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RELATIONS AT A COMBINED CONCENTRATION DISCONTINUITY IN A GAS CONTAINING SOLID PARTICLES

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A study is made of the flow of a mixture of gas and solid particles having discontinuities in the volume particle concentration m_2 when the gas flows through the discontinuities (combined discontinuities). There is a difficulty in describing such flows in that the conditions for using the two-liquid model $Z \gg l$ are not obeyed at the discontinuities, where Z is the characteristic scale in the change in mean flow parameters. This difficulty has been avoided [1] by replacing the region of discontinuity by a surface of discontinuity. With slight changes, this idea has been reproduced in all subsequent studies on combined discontinuities [2-8]. The continuous changes in gas parameters over the thickness of the discontinuity (much greater than the distance between the gas molecules) is replaced by a discontinuity of the first kind. A consequence of this is a physically unjustified increase in the entropy at the discontinuity [6], i.e., $[S] \sim [p]$. The physically correct conditions at the discontinuity were first used in [9] for the interaction of a shock wave with a porous half-space and a porous coating. Here the relations at the discontinuity have been derived on the assumption that the entropy is conserved when the gas flows into the porous material together with a Bord shock scheme for flow from it. These concepts were developed in [4, 10], where the surface force was introduced at the surface of discontinuity in the porosity, which acts on the gas and whose magnitude is chosen from the condition for the occurrence of given flow states at such discontinuities, which enables one to avoid the above entropy paradox. Similar concepts were partially used in [7] for the two-liquid model, where a surface force was artificially introduced that acts on particles at the combined-discontinuity surface.

Here we derive the relations at a combined discontinuity from the equations describing the flow of the gas at a discontinuity, which is an N-couple region, where N is the number of particles in the discontinuity. We calculate the surface force exerted by the gas on the particles.

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Consider the flow of a gas containing solid particles with a discontinuity in the volume particle concentration.

<u>Definition 1.</u> By discontinuity in m_2 we understand a region with transverse dimension h of the order of the distance between particles l in which there is a considerable change in the volume concentration $\Delta m_2 \sim m_2$; the motion is subsonic as regards the difference in the phase velocities, $M < 1$, where $M = |u_1 - u_2|/a_0$, and a_0 is the speed of sound in the gas. We take the surface of discontinuity as planar and direct the x axis perpendicular to it. The problem is to determine the gas parameters u_1, ρ_{11}, p, and T (velocity, density, pressure, and temperature) on one side of the discontinuity from the known ones on the other side. Particles at the discontinuity are acted on by a surface force, which leads to changes in the speed of the discontinuity, the number of particles in the discontinuity region, and the particle distribution in space.

<u>Definition 2.</u> The speed of the discontinuity is given by

$$D = \int\limits_{x_d - h/2}^{x_d + h/2} \langle m_2 u_2 \rangle \, dx \Bigg/ \int\limits_{x_d - h/2}^{x_d + h/2} \langle m_2 \rangle \, dx,$$

where x_d is the coordinate of the center of the discontinuity (Fig. 1), m_2 and u_2 are the volume content and velocity of the particles, and <f> is the mean value in the section x. It will be shown below that the characteristic time for the displacement of a particle by $\Delta x \approx d$ because of the surface force is $\tau \sqrt{(\rho_{22}/\rho_{11})}\,[d/(u_1 - D)]$ and, therefore, the Struhal number is $Sh \approx \sqrt{\rho_{11}/\rho_{22}}$. As in most cases, the density of the gas is much less than that of the particles; we have $Sh \ll 1$, so the gas motion in the discontinuity system may be taken as quasistationary [11].

We transfer to a frame of reference moving with velocity D and use the integral relations of [3], which in our case take the form

$$\int\limits_{\Sigma} \rho_{11} v_n d\sigma = 0, \quad \int\limits_{\Sigma} \rho_{11} \mathbf{v} \cdot v_n d\sigma = \int\limits_{\Sigma} \mathbf{p}_n d\sigma, \tag{1}$$

$$\int\limits_{\Sigma} \rho_{11} \left(\frac{v^2}{2} + U \right) v_n d\sigma = \int\limits_{\Sigma} \mathbf{p}_n \cdot \mathbf{v} d\sigma.$$

The relation between the coordinates in the laboratory system and the discontinuity system is $x_l = x + \int\limits_0^t D dt$, where Σ is a control surface, $\mathbf{v} = \mathbf{u}_1 - \mathrm{D}$, $\mathrm{D} = \mathrm{D} \cdot \mathbf{e}$, and $\mathbf{e}$ is a unit vector directed along the normal to S.

For simplicity we restrict ourselves to the one-dimensional approximation and put $m_2 = 0$ on the right and $m_2 > 0$ on the left. We place the origin at point 0 as shown in Fig. 1, where m_2 = const for $x < -h/2$ and $m_2 = 0$ for $x > h/2$. The gas parameters in the section x =

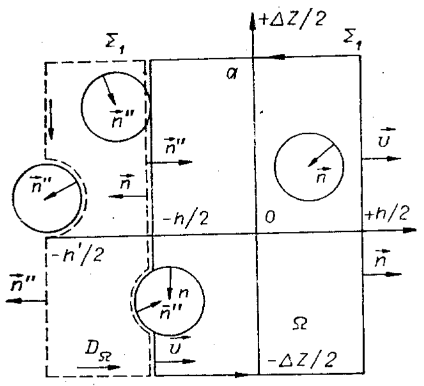

Fig. 1

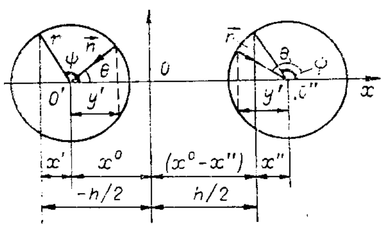

Fig. 2

$-h/2$ are denoted by f^-, while those in the section $x = +h/2$ are denoted by f^+. The region of discontinuity Ω is bounded by areas S perpendicular to the x axis and having coordinates $x_1 = -h/2$ and $x_2 = +h/2$, and at the points of intersection of an area with a particle it is bounded by part of the surface of the intersected particle G_1 (Figs. 1 and 3). The gas stream lines will form the lateral surface. We choose S such that it intersects a sufficiently large number of particles in the section $x_1 = -h/2$. The discontinuity region is an N-coupled one. As the transverse dimension of the Ω region is $\Delta z \gg l$, while the longitudinal one is $h \sim l$, the contribution to the integrals of (1) from particles intersecting the side surface can be neglected, while the stream lines are taken as parallel to the x axis. The condition $(\mathbf{n}\cdot\mathbf{v}) = 0$ is obeyed at the surfaces of the particles, so the presence of N particles in Ω has any effect only in the calculation of the second integral in (1). We project the second equation of (1) on the x axis on the basis that

$$\mathbf{p}_n = -p\cdot\mathbf{n},\ v_n = \mathbf{v}\cdot\mathbf{n},\ \mathbf{p}_n\cdot\mathbf{I}_x = -pn_x,$$

to get

$$\int_\Sigma \rho_{11} v v_n d\sigma = -\int_\Sigma p n_x d\sigma,$$

so

$$\int_{S^0} \rho_{11} v^2 dS\,|_{+h/2} - \int_{S^0} \rho_{11} v^2 dS\,|_{-h/2} = -\int_{S^0} p dS\,|_{+h/2} + \int_{S^0} p dS\,|_{-h/2} - \sum_N \int_\sigma p n_x d\sigma, \tag{2}$$

where S^0 is the area in the sections $\pm h/2$ through which the gas flows with $v_n \neq 0$. Formula (2) shows that the particles exert the force $F^0 = -\sum_N \int_\sigma p n_x d\sigma$ on the gas, so the average force acting from one particle in the discontinuity is $f = -\left(\sum_N \int_\sigma p n_x d\sigma\right)\Big/ N$. This means that the discontinuity moves with an acceleration $g = f\Big/\left(\frac{\pi d^3}{6}\rho_{22}\right)$, and that the term $\int_{V_{11}} \rho_{11} g dV$ must be added to the second equation in (1) on transferring to the discontinuity system. As the number of particles in the discontinuity is $N = nV$, while $V_{11} = m_1 V_1$, we get $\int_{V_{11}} \rho_{11} g dV \approx \frac{\rho_{11} m_1}{\rho_{22} m_2} \sum_N \int p_n d\sigma$, where $m_2 = \pi n d^3/6$. This shows that the contribution from the inertial term can be neglected for $\rho_{11}/\rho_{22} \ll 1$ and m_2 not too small. We transfer from (2) to a relation between the average characteristics of the gas flow in the sections $\pm h/2$. The average value of the gas parameter f at point x is defined by

$$\langle f\rangle = \frac{1}{S_1^0} \int_{S_1^0} f dS,$$

where S^0_1 is the part of S occupied by the gas. We use the equality of the mean surface and mean volume quantities [8] and multiply (2) by 1/S and use

$$\frac{1}{S} \int_{S_1^0} f dS = m_1 \langle f\rangle,$$

to get

$$(\langle\rho_{11} v^2\rangle + \langle p\rangle)\,|_{+h/2} - m_1(\langle\rho_{11} v^2\rangle + \langle p\rangle)\,|_{-h/2} = -\frac{1}{S} \sum_N \int_\sigma p n_x d\sigma. \tag{3}$$

We transform the right side of this equation to

$$\frac{1}{S}\sum_{N}\int_{\sigma} pn_x d\sigma = \frac{1}{S}\sum_{x^0}\sum_{y^0,z^0}\int_{\sigma} pn_x d\sigma,$$

where $\sum_{y^0,z^0}$ is summation over particles whose centers have coordinates (x^0, y^0, z^0) and lie in the volume bounded by the S planes passing through the points x^0 and $x^0 + dx^0$, while $\sum_{x^0}$ is summation over these volumes. The value of dx^0 is chosen from the conditions $dx^0 \ll h$, $N^0(x^0) \gg 1$, where $N^0(x^0)$ is the number of particles whose centers lie in the volume $dV^0 = Sdx^0$. Clearly, if the first condition is obeyed and S is chosen appropriately, the second condition can also be satisfied. As one can transfer from integration over the surface of a particle to integration with respect to θ and φ, and as the integration with respect to φ is with limits $0 \ll \varphi \leqslant 2\pi$ while the limits for θ are dependent only on x^0 (Fig. 2), we have

$$\frac{1}{S}\sum_{x^0}\sum_{y^0,z^0}\int_{\sigma} pn_x d\sigma = \frac{1}{S}\sum_{x^0}\int_{\sigma} d\sigma n_x \sum_{y^0,z^0} p,$$

and then as $d\sigma = r^2 \sin\theta d\theta d\varphi$ we have

$$\frac{1}{S}\sum_{x^0}\int_{\sigma} d\sigma n_x \sum_{y^0,z^0} p\,(x^0, y^0, z^0, \theta, \varphi) = \frac{1}{S}\sum_{x^0}\int_{\sigma} d\theta r^2 \sin\theta\, 2\pi N^0(x^0)\, p^\sigma n_x, \tag{4}$$

where $p^\sigma = \frac{1}{2\pi N^0(x^0)}\sum_{y^0,z^0}\int_0^{2\pi} p\,(x^0, y^0, z^0, \theta, \varphi)\, d\varphi$ is the mean pressure at the surface of a particle at the point $x = x^0 + r\cos\theta$. We represent the particle concentration in the discontinuity as $n\Phi(x^0)$, where n is the particle concentration at $x^0 < -h/2 + d/2$, $\Phi(x^0) = 1$ for $x^0 \leq -h/2 + d/2$, and $\Phi(x^0) = 0$ at $x^0 > h/2 - d/2$, which gives $N^0(x^0) = Sn\Phi(x^0)dx^0$. We replace summation with respect to x^0 by integration on the basis that $n_x = -\cos\theta$ to get (Fig. 2) that

$$-\frac{1}{S}\sum_{N}\int_{\sigma} pn_x d\sigma = \int_{-h/2-d/2}^{-h/2+d/2} dx^0 n\Phi(x^0)\int_0^{\psi} 2\pi r^2 p^\sigma (x^0 + y')\cos\theta\sin\theta d\theta + \int_{-h/2+d/2}^{+h/2-d/2} dx^0 n\Phi(x^0)\int_0^{\pi} 2\pi r^2 p^\sigma (x^0+y')\cos\theta\sin\theta d\theta, \tag{5}$$

where $\cos\theta = y'/r$, $\cos\psi = -x'/r$, $x' = x^0 + h/2$. Here the first integral ($-h/2 - d/2 \leq x^0 \leq -h/2 + d/2$) incorporates particles intersecting the area S in the section $x = -h/2$, while the second incorporates those lying in that region. It can be shown that the integrals with respect to θ become

$$\int_0^{\psi} 2\pi r^2 p^\sigma (x^0 + y')\cos\theta\sin\theta d\theta = \frac{\pi d^2}{2}\int_{-\left(\frac{x^0+h/2}{d/2}\right)}^{1} p^\sigma (x^0 + zr)\, zdz,$$

$$\int_0^{\pi} 2\pi r^2 p^\sigma (x^0 + y')\cos\theta\sin\theta d\theta = \frac{\pi d^2}{2}\int_{-1}^{1} p^\sigma (x^0 + zr)\, zdz.$$

In (5) we make the change of variable $y = (x^0 + h/2)/(d/2)$, $\cos\psi = -y$, $x^0(y) = yd/2 - h/2$ and use the definition $m_2^- = \pi nd^3/6$ to get

$$-\frac{1}{S}\sum_{N}\int_{\sigma} pn_x d\sigma = \frac{3}{2} m_2^- \left(\int_{-1}^{1} dy\Phi(x^0(y))\int_{-y}^{1} zp^\sigma\left(x^0(y) + \frac{zd}{2}\right)dz + \frac{2}{d}\int_{(-h/2+d/2)}^{(h/2-d/2)} dx^0\Phi(x^0)\int_{-1}^{1} p^\sigma (x^0 + zd/2)\, zdz\right). \tag{6}$$

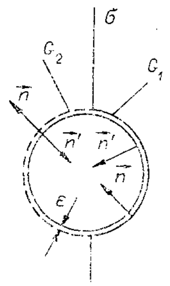

Fig. 3

Formulas (3) and (6) give the required relation at the discontinuity.

LEMMA. Relations (3) and (6) at the discontinuity are independent of the choice of the control surface Σ.

To prove this we take the control surface in the section $x = -h/2$ in such a way that the particles intersected by the area S lie entirely in Ω. Outside the particles, this surface coincides with that previously selected, while at the surface of the particles G_1 it is replaced by G_2 (Fig. 3). As $v_n = 0$ at the surface of a particle, the change in the surface of integration affects only the calculation of the integral over the surface of a boundary particle $\int pn_x d\sigma$, which in that case has the form

$$I = \int_{G_2} pn_x d\sigma + \int_{\sigma^0} pn'_x d\sigma,$$

where the second integral is taken over the entire particle surface (Fig. 3). As $\varepsilon \to 0$, the continuity of p means that the pressures at the surface of the particle and at the corresponding point on G_2 are identical, while $n'_x = -n_x$, so

$$\int_{\sigma^0} pn'_x d\sigma = \int_{G_1} pn_x d\sigma - \int_{G_2} pn_x d\sigma,$$

whence $I = \int_{G_1} pn_x d\sigma$, which proves the above assertion. It is clear that (3) and (6) are independent of the choice of the point x through which the control surface S passes because of the homogeneity in the mean gas and particle parameters. Let the control surface be taken in the section $-h'/2$, $h' \ll \Delta z$ (Fig. 1). As the flow is homogeneous, $m_1\langle\rho_{11}v^2 + p\rangle|_{-h/2} = m_1 \cdot \langle\rho_{11}v^2 + p\rangle|_{-h'/2}$, so it is necessary to show that the $\sum_N \int_\sigma pn_x d\sigma$ over the regions bounded by Σ_1 and Σ_2 coincide. We represent the integral over Σ_2 as a sum of integrals over Σ_1 and the surface D_Ω (dashed line in Fig. 1), while the integral over D_Ω is represented in the form of (4):

$$\sum_N \int_\sigma pn_x d\sigma = I_1 + I_2 + I_3, \quad I_1 = \sum_{x_1^0} \int_{\sigma_1} d\theta B(\theta, x^0),$$

$$I_2 = \sum_{x_2^0} \int_{\sigma_2} d\theta B(\theta, x^0), \quad I_3 = \sum_{x_3^0} \int_{\sigma^0} d\theta B(\theta, x^0),$$

where

$$B(\theta, x^0) = 2\pi r^2 \sin\theta n_x N^0(x^0) p^\sigma(x^0, \theta),$$

$$-h/2 - r \leqslant x_1^0 \leqslant -h/2 + r, \quad -h'/2 - r \leqslant x_2^0 \leqslant -h'/2 + r,$$

$$-h'/2 + r \leqslant x_3^0 \leqslant -h/2 - r.$$

Here there is summation over particles intersecting S in the section $-h/2$ in $\sum_{x_1^0}$, while in $\sum_{x_2^0}$ there is summation over particles intersecting S at $x = -h'/2$, and in $\sum_{x_3^0}$ there is summation over particles lying entirely in D_Ω. As the flow is homogeneous, we have for $-h'/2 \leqslant x \leqslant -h/2$ that

$$p^\sigma = \langle p\rangle + \Delta p^\sigma,\ \Delta p^\sigma(\theta) = \Delta p^\sigma(\pi - \theta),$$
$$\langle p\rangle = \text{const},\ N^0(x^0) = \text{const}.$$

As $n_x = -\cos\theta$, we get for the internal particles that

$$I_3 = -\sum_{x_3^0} N^0 2\pi r^2\left(\int_0^\pi \sin\theta\cos\theta d\theta\,\langle p\rangle + \int_0^\pi \sin\theta\cos\theta d\theta \Delta p^\sigma(\theta)\right) = 0$$

(the second term is zero because it is the integral of an odd function), and

$$I_1 = -2\pi r^2 N^0 \sum_{x_1^0}\int_\psi^\pi p^\sigma(\theta)\sin\theta\cos\theta d\theta,$$

$$I_2 = -2\pi r^2 N^0 \sum_{x_2^0}\int_0^\psi p^\sigma(\theta)\sin\theta\cos\theta d\theta.$$

In I_2, we replace summation with respect to x°_2 by x°_1 (this is possible because of the homogeneity) and get

$$I_1 + I_2 = -2\pi r^2 N^0 \sum_{x_1^0}\left(\langle p\rangle\int_0^\pi \sin\theta\cos\theta d\theta + \int_0^\pi \sin\theta\ \cos\theta\,\Delta p^\sigma d\theta\right).$$

Then, similarly, $I_3 = 0$ and we get $I_1 + I_2 = 0$, which proves the above assertion. Formulas (3) and (6) can be extended directly to the case $m_2 > 0$ for $x \geq h/2$, and then (3) takes the form

$$[m_1\langle\rho_{11}v^2 + p\rangle] = -\frac{1}{S}\sum_N\int_\sigma p n_x d\sigma. \tag{7}$$

If we add to the right side of (6) a contribution from the particles intersecting the area S in the section $+h/2$ and perform transformations analogous to those for particles intersecting S at $x = -h/2$, we get

$$-\frac{1}{S}\sum_N\int_\sigma p n_x d\sigma = \frac{3}{2}m_2^-\left(\int_{-1}^{1} dy\int_{-y}^{1} p^\sigma\left(x^0(y) + \frac{zd}{2}\right) z dz + \right. \tag{8}$$
$$\left. + \frac{2}{d}\int_{\left(-\frac{h}{2}+\frac{d}{2}\right)}^{\left(\frac{h}{2}-\frac{d}{2}\right)} dx^0 \Phi(x^0)\int_{-1}^{1} p^\sigma\left(x^0 + \frac{zd}{2}\right) z dz + \varkappa\int_{-1}^{1} dw\int_{-1}^{-w} p^\sigma\left(x^0(w) + \frac{zd}{2}\right) z dz\right),$$

where

$$\Phi(x^0) = 1,\ x^0 \leqslant -h/2 + d/2,\ \Phi(x^0) = \varkappa,\ x^0 \geqslant h/2 - d/2,$$
$$\varkappa = \text{const},\quad \varkappa \neq 1,\quad m_2^+ = \varkappa m_2^-,\quad m_2^- \neq 0,$$
$$w = (x^0 - h/2)/(d/2),\ x^0(w) = h/2 + wd/2.$$

We use identical transformations to represent (7) and (8) as

$$\langle m_1\rho_{11}v^2 + p\rangle^+ - \langle m_1\rho_{11}v^2 + p\rangle^- = \frac{3}{d}m_2^-\int_{-(h+d)/2}^{(h+d)/2}\Phi(x^0)\,dx^0 \times \tag{9}$$
$$\times\int_{-1}^{1} z p^\sigma\left(x^0 + \frac{zd}{2}\right)dz - \frac{3}{2}\int_{-1}^{1} dy\left[m_2^-\int_{-1}^{-y} z(\Delta p^\sigma)^- dz + m_2^+\int_{-y}^{1} z(\Delta p^\sigma)^+ dz\right].$$

As N (the connectedness of Ω) does not influence the calculation of the energy and mass integrals in (1), we neglect the work of the frictional force to get

$$[\langle m_1\rho_{11}v\rangle]=0,\quad \left[\left\langle H+\frac{v^2}{2}\right\rangle\right]=0, \tag{10}$$

where $[\varphi]=\varphi^+-\varphi^-$; $v=u_1-D$; and H is the specific gas enthalpy. Then (9) and (10) must be supplemented with the equation of state for the gas $p = p(\rho_{11}, T_1)$, $H = c_pT_1$.

We consider a particular case of (8) and (10) when

$$h=d,\quad D=0,\quad \langle\rho_{11}u_1^2\rangle=\langle\rho_{11}\rangle\langle u_1\rangle^2,$$
$$p^\sigma=\text{const},\quad \varkappa m_2^-=m_2^+,\quad \Phi(x^0)=\begin{cases}1, & x^0<0,\\ \varkappa, & x^0>0.\end{cases}$$

From (8) we get

$$-\frac{1}{S}\sum_N\int_\sigma pn_x d\sigma=p^\sigma[m_1],$$

and then from (10) we have

$$[(\langle p\rangle+\langle\rho_{11}\rangle\langle u_1\rangle^2)m_1]=p^\sigma[m_1]. \tag{11}$$

Expression (11) coincides with the formula given without derivation in [4], which dealt with the flow of a gas in a porous medium having surfaces of discontinuity in the porosity.

In [7], the following equations were derived for a surface of discontinuity from the two-liquid model:

$$\begin{gathered}[\rho_{11}m_1(u_1-D)]=0,\ [p+\rho_{11}m_1(u_1-D)^2]=0,\\ [H+(u_1-D)^2/2]=0,\ F_s=[m_2p],\end{gathered} \tag{12}$$

where F_s is the surface force acting on a particle at the discontinuity. If we neglect the pulsation terms in (9) and (10), the first and third equations in (12) derived in [7] coincide with (10) but Eq. (9) differs from the second equation in (12) in the presence of a right side. We take Δp^σ as introduced above as being the same as for the flow around a single particle [11], $\Delta p^\sigma = (1/2)\rho_{11}(x^0)(u_1(x^0) - D)^2(1 - (9/4)\sin^2\theta)$, while the pressure over the interval $-h/2 \le x \le h/2$ is interpolated from the formula $\langle p\rangle = (p^+ - p^-)(x/h) + (p^+ + p^-)/2$. We use (9) to get

$$[m_1\rho_{11}(u_1-D)^2+p]=m_2^-[p]+\frac{m_1^-}{m_1^+}\rho_{11}^-[m_1](u_1^--D)^2/2. \tag{13}$$

It can be shown that the two terms in (13) are the same in sign and order. Let $u^-_1 - D > 0$, $[m_1] > 0$; this case corresponds to the expansion of a current tube, which leads to an increase in p for a flow with $M < 1$ [11], so $[p] > 0$. The second assertion follows from the Bernoulli integral, which is conserved along the current tube. If the combined discontinuity is important, the condition $[m_2] \sim m_2 \sim 1$ is obeyed, and then it is necessary to incorporate the right side in (13). From (13) we get an estimate for the surface force in terms of the mean gas parameters in the form $F^0 \approx [m_1]\rho_{11}(u_1 - D)^2$, which gives the acceleration of a particle at the discontinuity as $g = -F^0/M^0$, where $h \approx d$, $M^0 = \rho_{22}m_2d$, and the characteristic time for the displacement of a particle by $\Delta x \approx d$ is $\tau\approx\sqrt{\frac{2\bar{m}_2\bar{\rho}_{22}}{[m_2]\rho_{11}}\frac{d}{(u_1-D)}}$, so the above hypotheses are obeyed.

In conclusion we note that, to determine p^σ, it is necessary to use experimental data on empirical relations such as those given in [4].

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AIR SHOCK HURLING OF AN UNFASTENED SOLID NEAR A FLAT OBSTACLE

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A formulation is given of the problem of a shock hurling a body near a solid flat obstacle. It is considered that the condition of a long shock interacting with the body [1] is satisfied and the force pattern is representable by two phases, diffraction and quasistationary streamlining. Initial conditions for the origination of different modes of motion, as well as the transient conditions associated with a variable mode during shock interaction with the obstacle, are considered for two versions of the diffraction load representation. The solution is obtained by a numerical method.

1. A body is considered that has a plane Ω of material symmetry in which forces from a shock and the reactions of unilateral constraints act, which corresponds to the plane-parallel motion of the body with variable (from 1 to 3) degrees of freedom. Let the plane Ω coincide with the inertial XOY coordinate system with origin at the body center of mass, which is symmetric relative to Y and with two points of contact with the obstacle for $t \le 0$ (Fig. 1). The shock is propagated along the X axis and is continuous with the body at $t = 0$. The unperturbed wave parameters are associated with the point $X = 0$.

It is assumed that the system of forces in the diffraction phase is independent of the body displacements, which are not substantial, while it is determined in the streamlining phase by stationary aerodynamics relationships in which the time t is a parameter [1]. Collision of the body support with the obstacle is considered absolutely inelastic while the resistance to displacement is subject to Coulomb's law. Four modes of motion are possible: 1) ($E = 1$) rotation in combination with slip along the obstacle; 2) ($E = 2$) rotation around a fixed axis; 3) ($E = 3$) slip; 4) ($E = 4$) flight without contact with the obstacle. Mode alternation is allowable during the motion. The criterion $E = 0$ is introduced for the state of rest.

2. The load in the diffraction phase can be approximated by an instantaneous impulse or function of time, which is important to estimation of the body acceleration during its most intensive loading. It is considered that the impulses S_W, S_A, and the moment of the impulse M_S are known along the X, Y axes. We then write the approximate expressions for the frontal force W, the lift force A, and the moment M_0 for the second case.

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